What might be known – Epistemic modality and uncertain contexts

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1 Epistemic modality in assertions and questions

(1) Hitch might be the culprit.

(2) Might Hitch be the culprit?

How to make (1) informative in dynamic semantics?
How to make (2) inquisitive? How to formalize its meaning?

2 Formal background

2.1 Epistemic modality

Kratzer (1981, 1991)

A modal sentence quantifies over possible worlds: $\mathsf{Q}(B)(\varphi)$
B: the modal base, restriction of the quantification.
φ: the prejacent proposition
Q: a quantifier = the modal force

(3) a. $\Diamond(B)(\varphi)$ is true in w iff $[\![B]\!]^w \cap [\![\varphi]\!] \neq \varnothing$ (consistency with B)
b. $\Box(B)(\varphi)$ is true in w iff $[\![B]\!]^w \subset [\![\varphi]\!]$ (entailment from B)

2.2 Dynamic semantics

Heim (1992); Groenendijk et al. (1996)

The meaning of an expression is its *Context Change Potential* (noted $[\![\cdot]\!]^{\text{ccp}}$).

(4) *Update* of s_i by φ: $s_i[\![\varphi]\!]^{\text{ccp}} = s_o$ (input context $\mapsto$ output context)

Formally a context will be implemented as an *information state*, viz. a set of possible worlds (f.t.s.o. simplification). For instance the CCP of a declarative sentence φ is:

(5) $s[\![\varphi]\!]^{\text{ccp}} = s \cap [\![\varphi]\!] = \{w \in s \mid [\![\varphi]\!]^w = 1\}$

2.3 Update Semantics for epistemic modality

Groenendijk et al. (1996), von Fintel and Gillies (2007)

Let s be an information state ($s \subset \mathcal{W}$).

(6) a. $s[\![\Diamond\varphi]\!]^{\text{ccp}} = \{w \in s \mid s[\![\varphi]\!]^{\text{ccp}} \neq \varnothing\} = \begin{cases} s \text{ if } s \cap [\![\varphi]\!] \neq \varnothing \\ \varnothing \text{ otherwise} \end{cases}$

b. $s[\![\Box\varphi]\!]^{\text{ccp}} = \{w \in s \mid s[\![\varphi]\!]^{\text{ccp}} = s\} = \begin{cases} s \text{ if } s \subset [\![\varphi]\!] \\ \varnothing \text{ otherwise} \end{cases}$

Contexts s are assimilited to epistemic modal bases.

2.4 Questions

Groenendijk and Stokhof (1984, 1989)

The denotation of a question w.r.t. w is the meaning of its complete true answer in w.

The meaning of a question is an equivalence relation on $\mathcal{W}$.

(7) a. $[\![?\varphi]\!]^w = \begin{cases} [\![\varphi]\!] & \text{if } [\![\varphi]\!]^w = 1 \\ [\![\neg\varphi]\!] & \text{otherwise.} \end{cases}$ or: $[\![?\varphi]\!]^w = \lambda w' \, ([\![\varphi]\!]^{w'} = [\![\varphi]\!]^w)$

b. $[\![?\varphi]\!] = \{\langle w, w'\rangle \in \mathcal{W} \times \mathcal{W} \mid [\![\varphi]\!]^{w'} = [\![\varphi]\!]^w\}$

Inquisitiveness of $?\varphi$ in s (Groenendijk, 1999):

(8) $?\varphi$ is inquisitive w.r.t s iff there exist w_1 and w_2 in s s.t. $[\![\varphi]\!]^{w_1} \neq [\![\varphi]\!]^{w_2}$.

I.e. s is consistent with φ and $\neg\varphi$ (as long as $?\varphi$ is a polar question).

3 Issues

An epistemic information state is based upon an epistemic *transitive/euclidean*[1] accessibility relation R_ε. Thus all worlds in s are accessible to each other.

(9) $\forall w, w' \in s, \; w \; R_\varepsilon \; w'$

Declaratives with epistemic modality do not add information to the context, as they leave it unchanged (cf. § 2.3).

Epistemic modal questions cannot be inquisitive:

(10) Let s be the speaker's epistemic information state;

a. $?\Diamond\varphi$ is inquisitive w.r.t. $s \Rightarrow s$ is consistent with $\Diamond\varphi$ *and* $\neg\Diamond\varphi$;

b. s is consistent with $\Diamond\varphi \Rightarrow$ there exists $w_1 \in s$ s.t. $\Diamond\varphi$ is true in w_1;

c. $\Diamond\varphi$ is true in $w_1 \Rightarrow$ there exists w_2 s.t. $w_1 \; R_\varepsilon \; w_2$ and φ is true in w_2;

d. φ is true in w_2 and (9) $\Rightarrow \Diamond\varphi$ is true in every world in s;

e. i.e. $\neg\Diamond\varphi$ is true in no world in $s \Rightarrow s$ is not consistent with $\neg\Diamond\varphi \Rightarrow \bot$.

4 Structuring information states

4.1 Information "spaces"

Let S be a *set* of information states ($S \subset \wp(\mathcal{W})$)

(11) CCP of modal sentences

a. $S[\![\Diamond\varphi]\!]^{\text{ccp}} = \{s \in S \mid s[\![\Diamond\varphi]\!]^{\text{ccp}} = s\} = \{s \in S \mid s \cap [\![\varphi]\!] \neq \varnothing\}$

b. $S[\![\Box\varphi]\!]^{\text{ccp}} = \{s \in S \mid s[\![\Box\varphi]\!]^{\text{ccp}} = s\} = \{s \in S \mid s \subset [\![\varphi]\!]\}$

(12) General case

$S[\![\psi]\!]^{\text{ccp}} = \{s' \mid \exists s \in S, \; s[\![\psi]\!]^{\text{ccp}} = s'\}$

4.2 Consistency and Support

Groenendijk et al. (1996)

In standard Update Semantics (simplified):

(13) a. φ is *consistent* with s iff $s[\![\varphi]\!]^{\text{ccp}}$ exists and $s[\![\varphi]\!]^{\text{ccp}} \neq \varnothing$.

b. φ is *supported* by s iff $s[\![\varphi]\!]^{\text{ccp}}$ exists and $s[\![\varphi]\!]^{\text{ccp}} = s$.

[1] Actually it is commonly assumed to be an equivalence relation (being reflexive too).

With respect to information spaces S:

(14) a. φ is *consistent* with S iff $S[\![\varphi]\!]^{\text{ccp}}$ exists and $S[\![\varphi]\!]^{\text{ccp}} \neq \varnothing$.
b. φ is *supported* by S iff $S[\![\varphi]\!]^{\text{ccp}}$ exists and $S[\![\varphi]\!]^{\text{ccp}} = S$.
c. φ is *minimally supported* by S iff $S[\![\varphi]\!]^{\text{ccp}}$ exists and there is at least an $s \in S$ s.t. $s \in S[\![\varphi]\!]^{\text{ccp}}$.
d. φ is *maximally consistent* with S iff $S[\![\varphi]\!]^{\text{ccp}}$ exists and for every $s \in S[\![\varphi]\!]^{\text{ccp}}$, $s[\![\varphi]\!]^{\text{ccp}} \neq \varnothing$.

(15) $?\Diamond\varphi$ is inquisitive in S iff φ is consistent *but not maximally consistent* with S.

4.3 Information spaces and common ground

A information space adds structure to the common ground (CG). Let c be a *context set*, i.e.: $c = \bigcap CG$.

(16) $S \subseteq \wp(c)$ and $c = \bigcup S$

Each $s \in S$ is generated by a different accessibility relation.

4.4 Back to Kratzer (1981)

Ordering sources = sets of propositions to complement the modal base.
An ordering source o induces an order $\leq_o$ among worlds of any modal base.
Let $\min_{\leq_o}([\![B]\!]^w)$ be the (sub)set of worlds in $[\![B]\!]^w$ that come closest to $\bigcap o$.

(17) a. $[\![\Diamond(B,o)(\varphi)]\!]^w = 1$ iff $\min_{\leq_o}([\![B]\!]^w) \cap [\![\varphi]\!] \neq \varnothing$
b. $[\![\Box(B,o)(\varphi)]\!]^w = 1$ iff $\min_{\leq_o}([\![B]\!]^w) \subset [\![\varphi]\!]$

Several ordering sources imply several variants of the modal base, viz. several states.

4.5 Complex possibilities ans states

Adapted from Groenendijk et al. (1996)

A possibility = a tuple $\langle w, o, g\rangle$ where o is a set of propositions and g is an assignment.

(18) An information state σ is now a set of tuples $\langle w, o, g\rangle$.

(19) $\sigma^{\downarrow} = \{w \mid \exists o \exists g\, \langle w, o, g\rangle \in \sigma\}$ (collecting the worlds present in σ)

(20) $\sigma[\![\Diamond\varphi]\!]^{\text{ccp}} = \{\langle w, o, g\rangle \in \sigma \mid \min_{\leq_o}(\sigma^{\downarrow}) \cap [\![\varphi]\!]^g \neq \varnothing\}$

(21) $\sigma[\![\varphi]\!]^{\text{ccp}} = \{\langle w, o, g\rangle \in \sigma \mid [\![\varphi]\!]^{w,g} = 1\}$

Static[2] relational meaning of a non-modal question:

(22) $[\![?\varphi]\!]^{\sigma} = \{\langle\langle w, o, g\rangle, \langle w', o', g\rangle\rangle \in \sigma \times \sigma \mid [\![\varphi]\!]^{w,g} = [\![\varphi]\!]^{w',g}\}$
Sorts out the worlds w in σ.
= 'Are there any worlds in the context σ with respect to which φ is true?'

Static relational meaning of an epistemic modal question:

(23) $[\![?\Diamond\varphi]\!]^{\sigma} = \{\langle\langle w, o, g\rangle, \langle w', o', g\rangle\rangle \in \sigma \times \sigma \mid \min_{\leq_o}(\sigma^{\downarrow}) \cap [\![\varphi]\!]^g \neq \varnothing \Leftrightarrow \min_{\leq_{o'}}(\sigma^{\downarrow}) \cap [\![\varphi]\!]^g \neq \varnothing\}$
Sorts out the ordering sources o in σ.
= 'Are there any ordering sources in the context σ according to which the known facts are consistent with φ?'

[2] Assume that the intension of α w.r.t. a context σ is $[\![\alpha]\!]^{\sigma} = \sigma[\![\alpha]\!]^{\text{ccp}}$ (i.e. the output of the CCP).

For a simpler formulation, with S:

(24) $[\![?\Diamond\varphi]\!]^S = \{\langle s, s'\rangle \in S \times S \mid s \cap [\![\varphi]\!] \neq \varnothing \Leftrightarrow s' \cap [\![\varphi]\!] \neq \varnothing\}$
= 'Are there any information states in the context S that are consistent with φ?'

5 Conclusion

Epistemics require the context to be structured as a set of information states. This can be achieved by supplying a traditional common ground with several sets of propositions handled as ordering sources.

As a consequence, questions with epistemics can be formalized, and they turn out to be questions about how is the context (and not merely about how is the world).

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